

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS  
APPLICATIONS  
UNITS 3&4

**SOLUTIONS**

Section Two:  
Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (5 marks)

The table below shows the duration and immediate predecessors for all the activities required to complete a project.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activity |  |  |  |  |  |  |
| Duration (minutes) |  |  |  |  |  |  |
| Immediate predecessors |  |  |  |  |  |  |

(a) Construct a project network to show all the above information. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ network with correct precedence relations  ü uses dummy edge correctly  ü adds direction and labels to edges |

(b) Determine the minimum completion time for the project and list, in order, the activities that lie on the critical path. (2 marks)

|  |
| --- |
| Solution |
| Minimum completion time is minutes.  Activities on critical path: . |
| Specific behaviours |
| ✓ minimum completion time  ü ordered list of activities on critical path |

Question 10 (9 marks)

The body length and eye diameter of eight tropical fish of the same species are shown in the table below.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Body length, cm |  |  |  |  |  |  |  |  |
| Eye diameter, mm |  |  |  |  |  |  |  |  |

(a) Use your calculator to graph the above data and hence describe the direction and strength of the linear association between the variables. (1 mark)

|  |
| --- |
| Solution |
| The association is strong and negative. |
| Specific behaviours |
| ✓ states strong and negative |

(b) Determine the coefficient of determination for the linear association and interpret its value in context. (2 marks)

|  |
| --- |
| Solution |
| of variation in the eye diameter can be explained by the variation in body length. |
| Specific behaviours |
| ✓ , to at least decimal places  ü correct interpretation of coefficient |

(c) Determine the equation of the least-squares line to predict from and interpret the slope of the line in context. (3 marks)

|  |
| --- |
| Solution |
| The slope means that for every cm increase in length of a fish, the eye diameter is expected to decrease by mm. |
| Specific behaviours |
| ✓ equation, using given variables  ü indicates as one variable increases, the other decreases  ü quantifies relationship, using units |

(d) Predict the eye diameter of another tropical fish of the same species that has a body length of cm and give two reasons that support the validity of this prediction. (3 marks)

|  |
| --- |
| Solution |
| Prediction is valid because correlation is strong, and it does not involve extrapolation. |
| Specific behaviours |
| ✓ calculates diameter  ü states strong correlation  ü states interpolation or no extrapolation |

Question 11 (7 marks)

A builder bought a scissor lift for and for accounting purposes will depreciate its value by annually.

(a) Show that the value of the lift after one year will be . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ clearly shows any correct method |

(b) Write a recursive rule to calculate the value of the lift, in dollars, after years.

(2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ recursive part  ü initial term using |

(c) Calculate the value of the lift after years. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value (or rounded to nearest $) |

(d) Deduce the term rule for the value of the lift after years. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct rule |

(e) Determine the least number of years that must pass for the lift to depreciate by more than of its original value. (2 marks)

|  |
| --- |
| Solution |
| of value will remain, so value will be .  Least number of years is . |
| Specific behaviours |
| ✓ indicates required value  ü correct number of years |

Question 12 (7 marks)

A reducing balance loan is defined by the recurrence relation , where is the balance of the loan in dollars at the start of month . The relation was used to create the following spreadsheet.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Month | Balance of loan at start of month | Monthly interest | Monthly repayment | Loan balance carried forward |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

(a) Determine the annual percentage interest rate that applies to the loan. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates monthly rate  ü correct annual rate as percentage |

(b) State the value of each of the constants and in the recurrence relation. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of  ü value of and value of |

(c) Determine the value of , the value of and the value of shown in the spreadsheet.

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of  ✓ value of  ✓ value of |

Question 13 (8 marks)

A researcher observed a large number of cats, noting for each one its gender and the paw that it used to react to a stimulus placed in front of it. The numbers in each category are shown in the table below.

|  |  |  |
| --- | --- | --- |
| Cats | Left-pawed | Right-pawed |
| Male |  |  |
| Female |  |  |

The researcher is interested in whether there is an association between the variables.

(a) Name one of the variables in the study and classify it as numerical or categorical.

(2 marks)

|  |
| --- |
| Solution |
| Name: Gender or Paw.  Classifies as Categorical. |
| Specific behaviours |
| ✓ names one variable  ü classifies variable |

(b) Determine the percentage of left-pawed cats that were female. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct total for category  ü correct percentage (whole number ok) |

(c) Use the above data to complete the following table so that it shows column percentages rounded to the nearest whole number. (2 marks)

|  |
| --- |
| Solution |
| See table |
| Specific behaviours |
| ü both columns add to 100  ✓ correct table |

|  |  |  |
| --- | --- | --- |
| Cats (%) | Left-pawed | Right-pawed |
| Male |  |  |
| Female |  |  |

(d) Explain whether the percentaged table suggests the presence of an association between the variables. (2 marks)

|  |
| --- |
| Solution |
| An association is evident between the variables as the pairs of percentages in each row (i.e., for each gender) are quite different. |
| Specific behaviours |
| ✓ states association  ü explanation using different percentages |

Question 14 (9 marks)

The table below shows the number of new clients attracted by a business during the first three weeks of an advertising campaign, together with some derived values to assist in the analysis of the time series data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Week | Day | | New Clients | Weekly mean | Percentage of weekly mean |
|  | Mon |  |  |  |  |
| Tue |  |  |  |
| Wed |  |  |  |
| Thu |  |  |  |
| Fri |  |  |  |
|  | Mon |  |  |  |  |
| Tue |  |  |  |
| Wed |  |  |  |
| Thu |  |  |  |
| Fri |  |  |  |
|  | Mon |  |  |  |  |
| Tue |  |  |  |
| Wed |  |  |  |
| Thu |  |  |  |
| Fri |  |  |  |

(a) Calculate the value of , the value of and the value of in the table. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of  ✓ value of  ✓ value of |

(b) Determine the seasonal index for Thursdays. (2 marks)

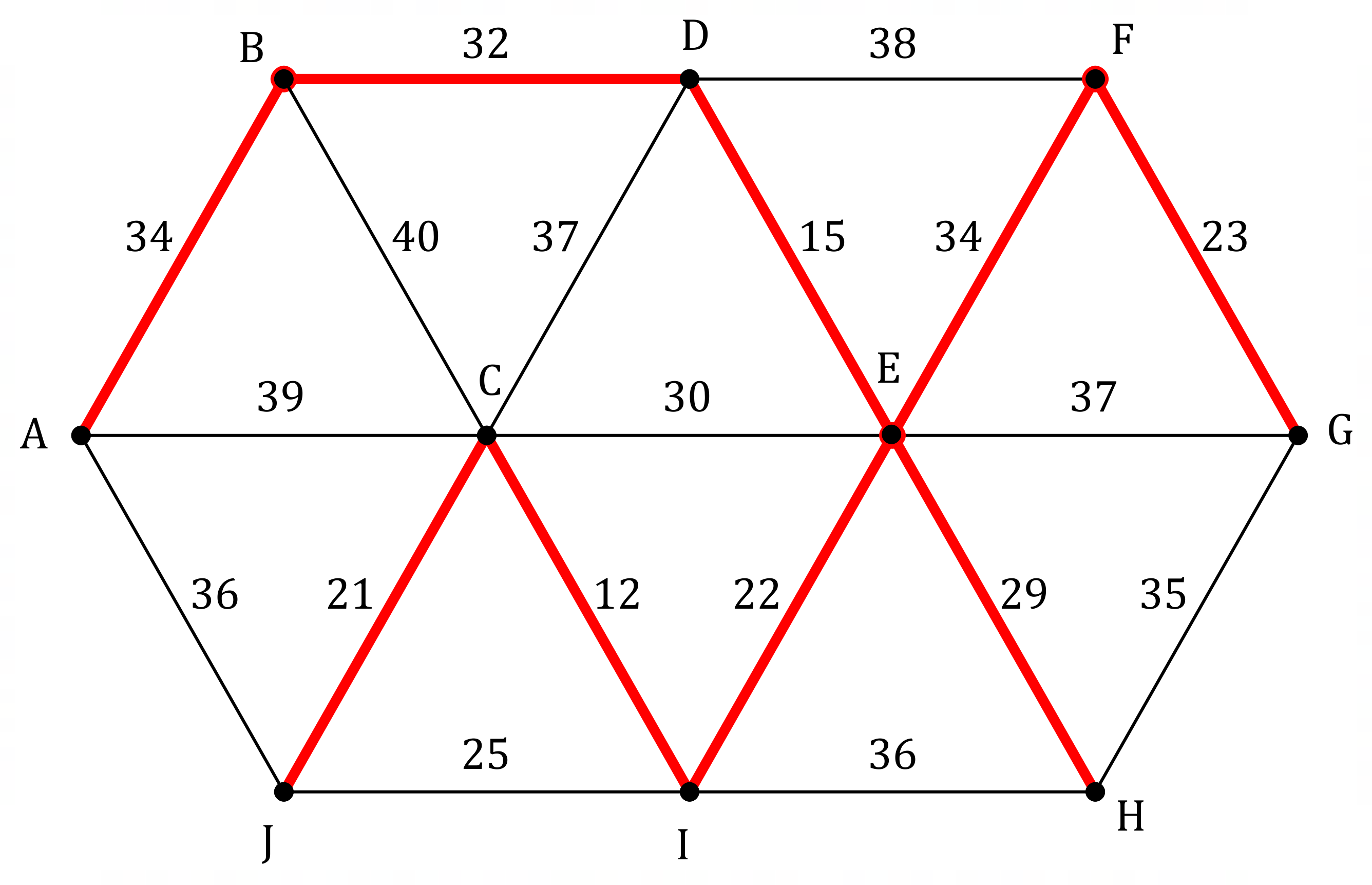
|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses correct indices  ü calculates index as percent or decimal |

(c) The least-squares line to predict the deseasonalised number of new clients from the day is . Stating any assumptions made, determine the best estimate for the number of new clients expected on Wednesday of Week 4. (4 marks)

|  |
| --- |
| Solution |
| Wednesday of Week is day .  Expect new clients, assuming that the existing trend and seasonality continues into the future. |
| Specific behaviours |
| ✓ uses correct day to calculate deseasonalised number  ü calculates seasonal index for Wednesday  ü uses index to calculate whole number estimate  ü states assumption |

Question 15 (8 marks)

Ten mains-powered smoke alarms must be installed in a building. The edge weights in the graph below represent the length of cable, in metres, required between adjacent alarms.



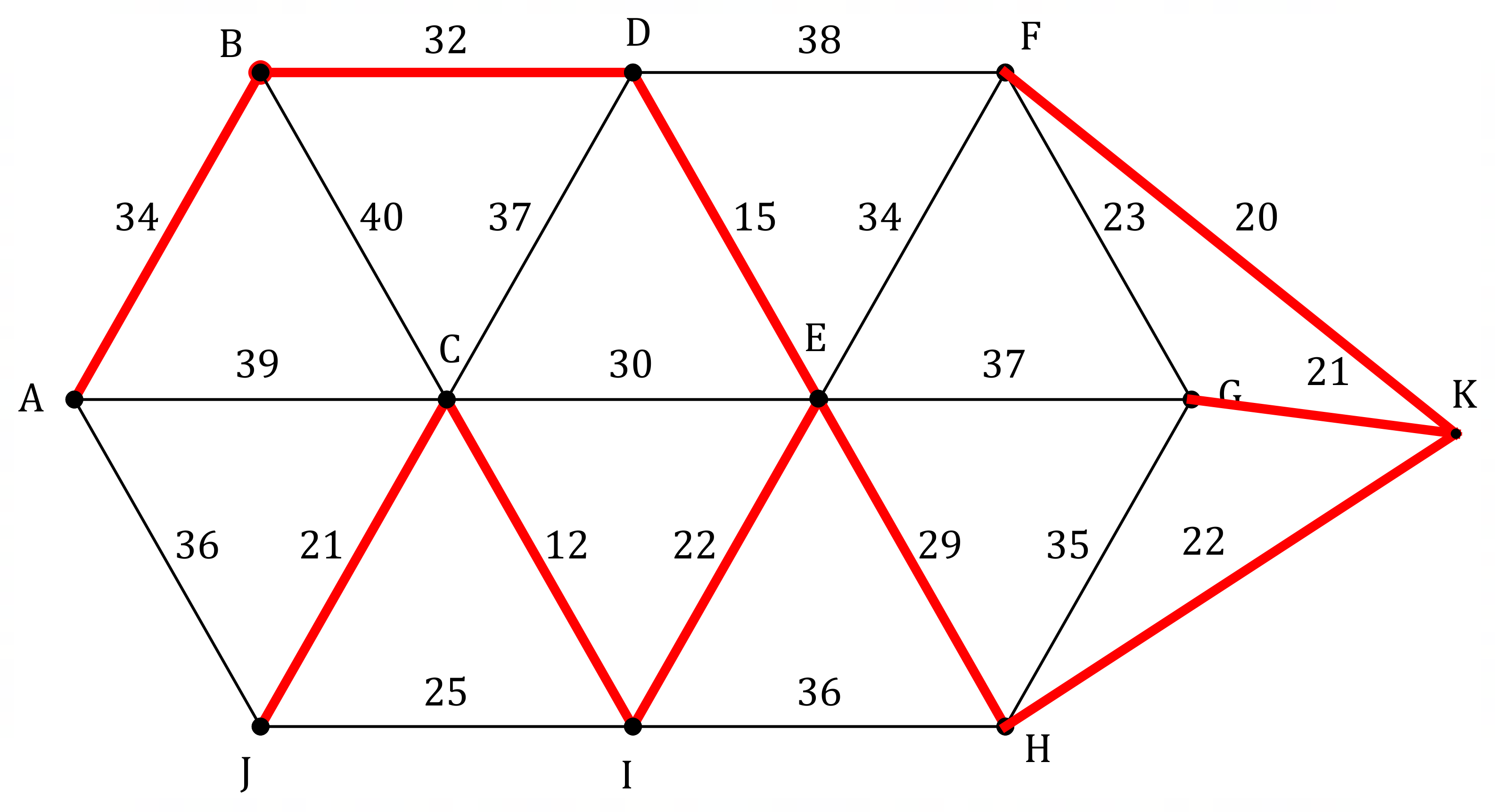
(a) Clearly identify the minimum spanning tree on the graph above. (3 marks)

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ any tree; ü any spanning tree; ü minimum spanning tree |

(b) Determine the cost of installing the cabling between the alarms using the minimum spanning tree, given that each metre of cabling will cost . (2 marks)

|  |
| --- |
| Solution |
| Sum of edges:  Total cost: |
| Specific behaviours |
| ✓ sum of edges  ü calculates cost |

(c) Explain how your answer to part (b) will change if smoke alarm is added to the system with cable lengths of and metres to alarms and respectively. (*A copy of the graph from the previous page is shown below if you wish to use it*.) (3 marks)



|  |
| --- |
| Solution |
| The minimum spanning tree changes so that edges and no longer used but edges and are now included.  The edge sum increases by to .  Total cost increases by to . |
| Specific behaviours |
| ✓ states change to edges  ü calculates increase in (or new) sum of edges  ü calculates increase in (or new) cost |

Question 16 (8 marks)

Saltwater flows steadily into a tank, where it is mixed with existing water. An overflow spout on the tank allows excess water to flow out. The salt concentration in the tank can be modelled by , where is the concentration, in parts per million, after saltwater has been flowing into the tank for minutes.

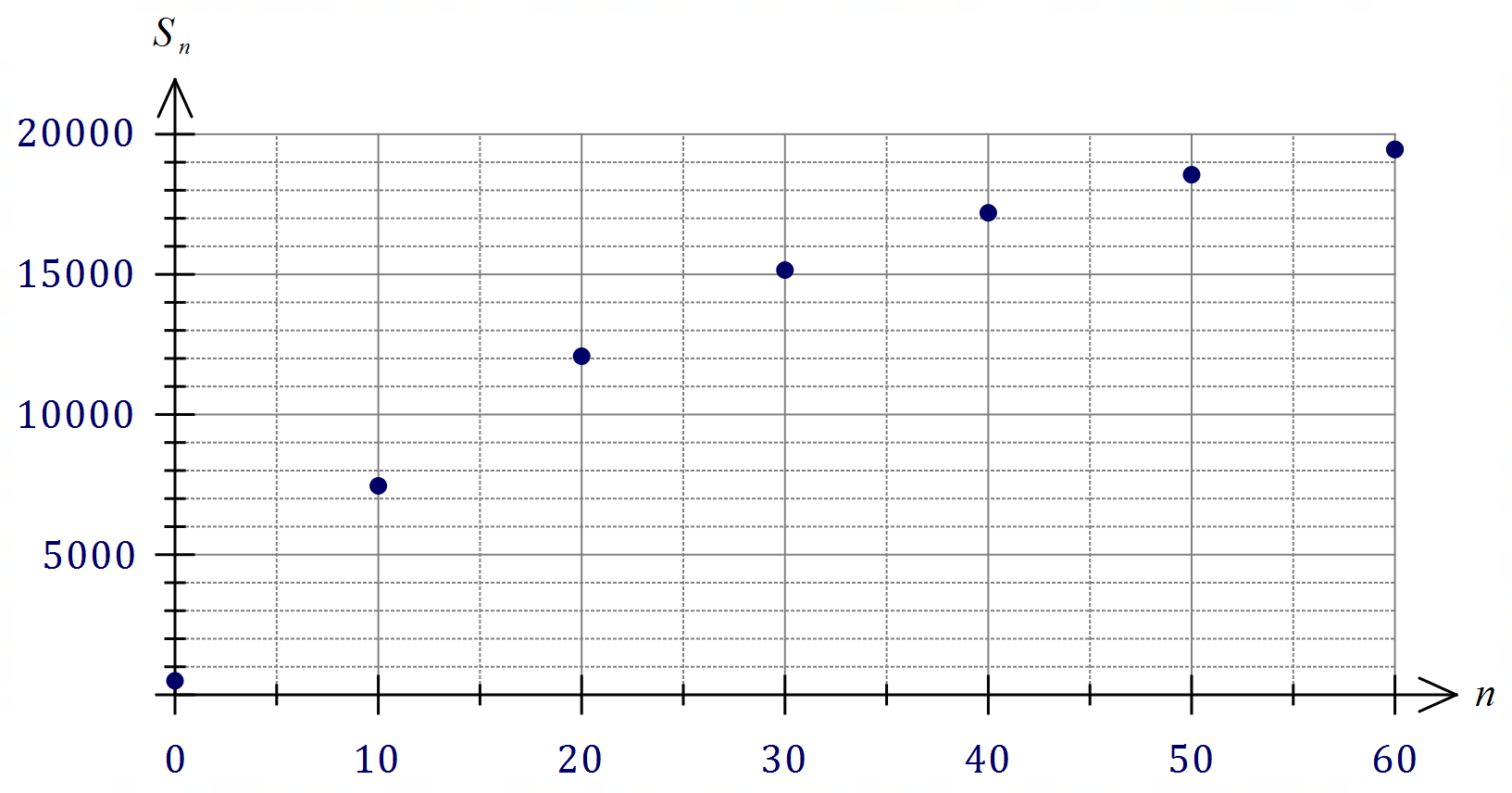
(a) Determine . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(b) Determine the value of for to first exceed ppm. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(c) Plot points to show the salt concentration at minute intervals on the axes below, after first adding an appropriate scale to the vertical axis. (3 marks)





|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ adds scale  ü at least 4 points  ü all points |

(d) Describe the feature of the plotted points that indicates the salt concentration will eventually reach a steady-state. (1 mark)

|  |
| --- |
| Solution |
| Describes 'levelling off' or 'increasing at a decreasing rate', etc. |
| Specific behaviours |
| ✓ reasonable description |

(e) Determine, with justification, the steady-state salt concentration. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ equation, or explanation using term(s) with large  ü correct concentration |

Question 17 (7 marks)

Loans are offered by lender at a rate of per annum compounded quarterly and from lender at a rate of per annum compounded daily.

(a) Calculate the total amount that must be repaid if is borrowed from lender for  
nine months. (2 marks)

|  |  |
| --- | --- |
| Solution | |
| Hence must be repaid. | Financial calculator  Solve for  Hence must be repaid. |
| Specific behaviours | |
| ✓ shows compound interest formula or values used in financial calculator  ü correct amount | |

(b) Calculate the total interest that will be charged on a loan of from lender for  
 days. (2 marks)

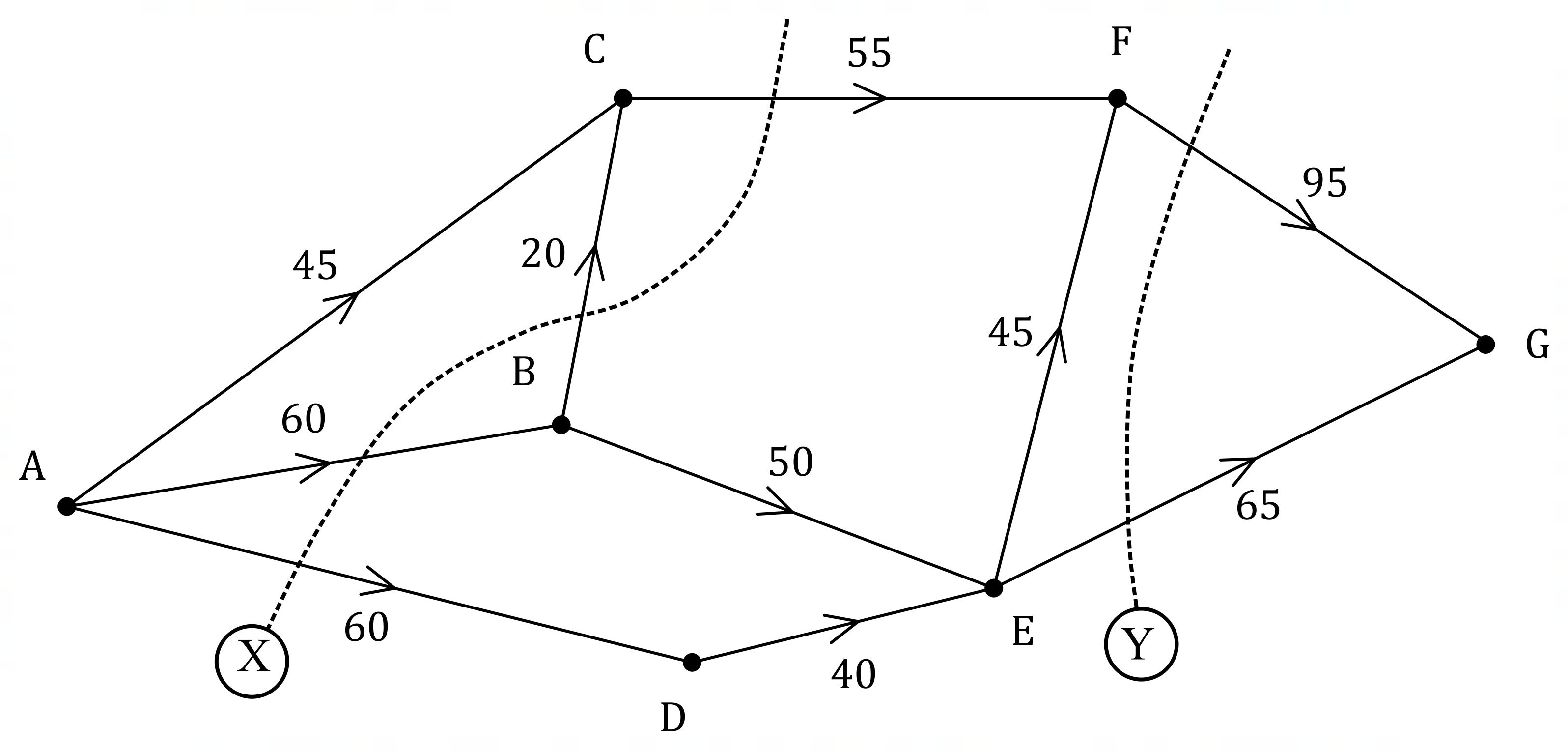
|  |  |
| --- | --- |
| Solution | |
| Hence interest charged. | Financial calculator  Solve for  Hence interest charged. |
| Specific behaviours | |
| ✓ shows compound interest formula or values used in financial calculator  ü correct amount of interest | |

(c) Minimising loan interest is the primary goal for a borrower. Calculate the effective interest rate for each lender and hence recommend which should be chosen. (3 marks)

|  |
| --- |
| Solution |
| Lender .  Lender .  Hence interest will be minimised by choosing lender  (as their effective interest rate is less than that of lender ). |
| Specific behaviours |
| ✓ effective interest rate for  ü effective interest rate for  ü uses effective rates to choose lender |

Question 18 (7 marks)

When a city bypass is closed, traffic that would normally use it is forced to flow through main roads in the city. The edge weights on the directed graph below show the maximum number of vehicles per minute that can travel between junctions (represented by vertices) without causing congestion in the city.



(a) Determine the value of cut and the value of cut . (2 marks)

|  |
| --- |
| Solution |
| Cut vpm. Cut vpm. |
| Specific behaviours |
| ✓ value of cut  ü value of cut |

(b) Determine the maximum flow of vehicles per hour from to . (3 marks)

|  |
| --- |
| Solution |
| .  OR  Minimum cut is .  Maximum hourly flow is vehicles per hour. |
| Specific behaviours |
| ✓ indicates systematic listing or several cuts  ü correct maximum flow per minute  ü correct maximum flow per hour |

(c) City engineers recommend taking steps to improve traffic flow between junctions and . Determine, with reasoning, the maximum increase in the hourly flow of vehicles from to that their plan could achieve. (2 marks)

|  |
| --- |
| Solution |
| Spare capacity along is , so increase by .  OR  Minimum cut not using is , so .  Maximum possible increase is vehicles/hour. |
| Specific behaviours |
| ✓ reasoning  ü correct increase |

Question 19 (8 marks)

A fund with a balance of is used to create an annuity, from which regular withdrawals of are to be made at the end of each quarter. Interest at a rate of per annum is added to the fund quarterly, just before each withdrawal.

(a) Use one or more calculations to show that the balance of the fund after one withdrawal  
is . (2 marks)

|  |  |
| --- | --- |
| Solution | |
|  |  |
| Specific behaviours | |
| ✓ indicates adjustment for interest for one quarter  ü calculation: interest less withdrawal or recursive type | |

(b) Write a recurrence relation to calculate the balance after the withdrawal. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct recursive part  ü correct statement for |

(c) Determine the total interest earned by the fund during its first year of operation. (2 marks)

|  |
| --- |
| Solution |
| Total interest is . |
| Specific behaviours |
| ✓ indicates closing balance  ü correct total interest |

(d) The amount of the regular withdrawal from the fund can be modified so that from the outset, the fund is a perpetuity rather than an annuity. Determine the withdrawal required for this to occur. (2 marks)

|  |
| --- |
| Solution |
| Withdrawal is interest for one quarter:  Amount of withdrawal is . |
| Specific behaviours |
| ✓ indicates withdrawal must be interest for one quarter  ü correct amount |

Question 20 (8 marks)

For each quarter in the years from to , a city library calculated its mean number of users per day and used seasonal indices to deseasonalise the data. A snapshot of the data for the year is shown in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Quarter |  |  |  |  |
| Mean daily users |  |  |  |  |
| Deseasonalised mean daily users |  |  |  |  |

The trend line for the deseasonalised mean daily users is where is the quarter and corresponds to the first quarter of .

(a) State, with justification, whether the mean number of users per day was increasing or decreasing over the years. (1 mark)

|  |
| --- |
| Solution |
| Decreasing - slope of trend line is negative |
| Specific behaviours |
| ✓ states decreasing, justifies with slope |

(b) Calculate the seasonal indices and enter them in the following table. (2 marks)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Quarter |  |  |  |  |
| Seasonal Index |  |  |  |  |

|  |
| --- |
| Solution |
| See table: Decimals to at least dp (or equivalent as percentage). |
| Specific behaviours |
| ✓ at least two correct and indices sum to  ü all correct |

(c) State, with justification, in which quarter the library was usually busiest during the period that data was collected. (2 marks)

|  |
| --- |
| Solution |
| Third quarter - the seasonal index is highest for this quarter. |
| Specific behaviours |
| ✓ correct quarter  ü justifies using indices |

(d) Determine an estimate, to the nearest whole number, for the mean daily users the library experienced in the first quarter of . (3 marks)

|  |
| --- |
| Solution |
| Estimate there were users. |
| Specific behaviours |
| ✓ correct value of  ü calculates  ü seasonalises to obtain correct estimate |

Question 21 (7 marks)

An electronics store advertises a TV for sale at a price of . Rather than pay this amount in full, the store offers customers a no deposit reducible balance loan with monthly payments of . The first repayment is due one month after the customer makes the purchase, just after interest for the month is added to the loan balance.

(a) Determine the total interest paid by customers who buy the TV using the loan. (2 marks)

|  |
| --- |
| Solution |
| Repay:  Interest: |
| Specific behaviours |
| ✓ calculates total repaid  ü calculates interest |

(b) Determine the annual percentage interest rate that applies to the loan. (2 marks)

|  |
| --- |
| Solution |
| Using Financial app:  Solve for , so rate is per annum. |
| Specific behaviours |
| ✓ shows correct values for financial calculator  ü correct annual rate |

A customer decides to buy the TV using a similar reducible balance loan to that offered by the store but financed by their bank at an annual interest rate of and over months.

(c) Determine their monthly repayment. (2 marks)

|  |
| --- |
| Solution |
| Using Financial app with:  Solve for , so their repayment is per month. |
| Specific behaviours |
| ✓ shows correct values for financial calculator  ü correct repayment |

(d) Determine the total interest that the bank will charge on the loan over the months.

(1 mark)

|  |
| --- |
| Solution |
| Using Financial app: .  Using .  Total of or in interest. |
| Specific behaviours |
| ✓ correct amount |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

© 2021 WA Exam Papers. Kennedy Baptist College has a non-exclusive licence to copy and communicate this document for non-commercial, educational use within the school. No other copying, communication or use is permitted without the express written permission of WA Exam Papers. SN245-186-4.